6.1 Part II Assignment

AP Stats

1. The probabilities that a customer selects 1, 2, 3, 4, or 5 items at a convenience store are 0.32, 0.12, 0.23, 0.18, and 0.15, respectively.

a. Construct a probability distribution (table) for the data, and verify that this is a legitimate probability distribution.

X	1	2	3	4	5
Prob	0.32	0.12	0.23	0.18	0.15

$\sum P(x) = 1$

$\therefore$ legitimate probability distribution

b. Find the mean of the random variable, X.

$$\mu_x = 1(0.32) + 2(0.12) + 3(0.23) + 4(0.18) + 5(0.15)$$
$$= \boxed{2.72}$$

c. Find the standard deviation of X.

$$\sigma_x^2 = (2.72-1)^2(0.32) + (2.72-2)^2(0.12) + (2.72-3)^2(0.23) + (2.72-4)^2(0.18) + (2.72-5)^2(0.15)$$
$$= 2.1016$$
$$\sigma_x = \sqrt{2.1016} = \boxed{1.45}$$

2. Here's a game: If a player rolls two dice and gets a sum of 2 or 12, he wins \$20. If the person gets a 7, he wins \$5. The cost to play the game is \$3. Find the expected payout for the game.

X = # WON	17	2	-3
Prob	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{28}{36}$

$$\text{EXPECTED PAY} = 17\left(\frac{2}{36}\right) + 2\left(\frac{6}{36}\right) - 3\left(\frac{28}{36}\right)$$
$$= -\$1.06$$

3. A distribution of grades in an introductory statistics class (where A = 4, B = 3, etc.) is:

X	0	1	2	3	4
p(X)	.10	.15	.30	.30	.15

a. A graduate student needs at least a B to get credit for the course. What are her chances of getting at least a B? $0.3 + 0.15 = 0.45$

b. Find $P(1 \le X < 3)$. $P(X = 1 \text{ or } 2) = 0.15 + 0.3 = 0.45$

c. Find the average (i.e., mean) grade in this class.

$$\mu_X = 0(0.1) + 1(0.15) + 2(0.3) + 3(0.3) + 4(0.15)$$
$$= 2.25$$

d. Find the standard deviation for the class grades.

$$\sigma_X^2 = (2.25-0)^2(0.1) + (2.25-1)^2(0.15) + (2.25-2)^2(0.3) + (2.25-3)^2(0.3) + (2.25-4)^2(0.15)$$
$$= 1.39$$
$$\sigma_X = \sqrt{1.39} = 1.18$$

e. Find the lowest grade X_0 such that $P(X \ge X_0) < 0.5$. $P(X \geq 3) = 0.45$

$\therefore X_0 = 3$

f. Circle the correct answer: X is an example of a (discrete) (continuous) random variable.

4. A lottery offers one \$1,000 prize, one \$500 prize, and five \$100 prizes. One thousand tickets are sold at \$3 each. Find the expectation (expected value) if a person buys one ticket.

\$ WIN	997	497	97	-3
P(win)	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{5}{1000}$	$\frac{993}{1000}$

$$\text{Expectation} = 997\left(\frac{1}{1000}\right) + 497\left(\frac{1}{1000}\right) + 97\left(\frac{5}{1000}\right) - 3\left(\frac{993}{1000}\right)$$
$$= -\$1.00$$