Kay

Calculus 12

3.4 Rates of Change

Recall :	Velocity is the rate of change of position with respect to time.
	This means that velocity is the derivative of position with respect to time.

Ex.1 The position of a particle that is moving in a straight line, measured in metres, is given by the equation $s = t^3 - 9t^2 + 24t,\ \ t \geq 0$, where t is the time in seconds.

a) Determine the velocity of the particle at a time t.

$v(t) = \frac{ds}{dt} = 3t^2 - 18t + 24$

b) Determine the velocity of the particle at t=3 seconds.

$v(3) = 3(3)^2 - 18(3) + 24 = -3$ m/s

c) When is the particle at rest?

Note: at rest $\Rightarrow v(t) = 0$

$3t^2 - 18t + 24 = 0$

$3(t^2 - 6t + 8) = 0$

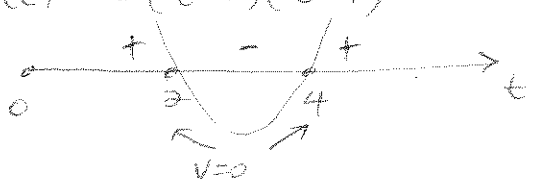
$3(t-2)(t-4) = 0 \quad \therefore t = 2, 4$

d) When is the particle moving in the positive/negative direction?

Note: moving forward $\Rightarrow v(t) > 0$

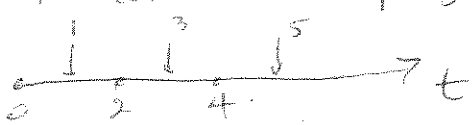
moving backward $\Rightarrow v(t) < 0$

$v(t) = 3(t-2)(t-4)$



$\therefore$ the particle is moving forward when $0 \leq t < 2$ or $t > 4$ and moving backward when $2 < t < 4$

Note: If $v(t)$ is not a polynomial, test a point in each interval.



$v(1) = 3(1-2)(1-4) > 0$

$v(3) = 3(3-2)(3-4) < 0$

$v(5) = 3(5-2)(5-4) > 0$

e) Draw a diagram to illustrate the movement of the particle.

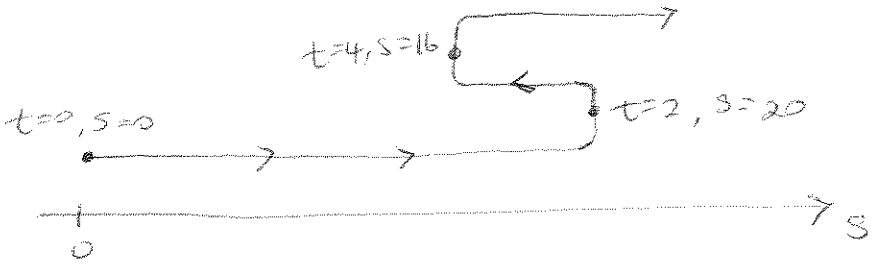
Find the position at $t=0$ and when the particle is at rest $(t=2,4)$

$s = f(t) = t^3 - 9t^2 + 24t$

$f(0) = 0$

$f(2) = 2^3 - 9(2)^2 + 24(2) = 20$

$f(4) = 4^3 - 9(4)^2 + 24(4) = 16$



f) Find the total distance travelled by the particle in the first 6 seconds.

Find the position at $t=6$

$f(6) = 6^3 - 9(6)^2 + 24(6) = 36$

Find the distance for each change of direction

$0 \le t \le 2$: $d = 20 - 0 = 20$

$2 \le t \le 4$: $d = |16 - 20| = 4$

$4 \le t \le 6$: $d = 36 - 16 = 20$

total distance $= 20 + 4 + 20 = \underline{44m}$

Practice Work: 3.4 #1, 3, 7, 8

Even answers: #8a) 17 m/s b) 5 seconds